GCE Examinations

Advanced Subsidiary / Advanced Level

Further Pure Mathematics
Module FP2

Paper H

MARKING GUIDE

This guide is intended to be as helpful as possible to teachers by providing concise solutions and indicating how marks should be awarded. There are obviously alternative methods that would also gain full marks.

Method marks (M) are awarded for knowing and using a method.

Accuracy marks (A) can only be awarded when a correct method has been used.

(B) marks are independent of method marks.



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FP2 Paper H – Marking Guide

1. $2x^2 + y^2 = 4,\ 4x + 2y\dfrac{dy}{dx} = 0$ or $2x + y\dfrac{dy}{dx} = 0$ — M1 A1

$2 + y\dfrac{d^2y}{dx^2} + \left(\dfrac{dy}{dx}\right)^2 = 0$ — M1 A1

at $(1, -\sqrt{2})$ $\quad 2 - \sqrt{2}\dfrac{dy}{dx} = 0 \ \therefore\ \dfrac{dy}{dx} = \sqrt{2}$ — A1

$2 - \sqrt{2}\dfrac{d^2y}{dx^2} + 2 = 0 \ \therefore\ \dfrac{d^2y}{dx^2} = 2\sqrt{2}$ — A1

$\rho = \dfrac{(1+(\sqrt{2})^2)^{\frac{3}{2}}}{2\sqrt{2}} = \dfrac{3\sqrt{3}}{2\sqrt{2}} = \dfrac{3\sqrt{6}}{4}$ — M1 A1 **(8)**

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2. *(a)* $f(x) = \cosh x = \frac{1}{2}(e^x + e^{-x})$

$f(-x) = \frac{1}{2}(e^{-x} + e^{-(-x)}) = \frac{1}{2}(e^{-x} + e^x) = f(x) \ \therefore\ \cosh x$ is even — M1 A1

(b) $\ln(xy) = \operatorname{arcosh}(\frac{5}{3}) = \ln\left(\frac{5}{3} + \sqrt{\frac{25}{9} - 1}\right) = \ln 3$ — M1 A1

$\therefore\ xy = 3$ — A1

$\cosh(3x - y) = 1 \ \therefore\ 3x - y = 0$ — B1

$\therefore\ 3x = y = \dfrac{3}{x},\ x^2 = 1$ — M1

$x > 0 \ \therefore\ x = 1, y = 3$ — A1 **(8)**

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3. $\displaystyle\int \frac{1}{13\cosh x - 5\sinh x}\,dx = \int \frac{1}{\frac{13}{2}(e^x + e^{-x}) - \frac{5}{2}(e^x - e^{-x})}\,dx$ — M1

$\displaystyle= \int \frac{1}{4e^x + 9e^{-x}}\,dx$ — A1

$\displaystyle= \int \frac{e^x}{4e^{2x} + 9}\,dx$ — M1

$u = e^x,\ \dfrac{du}{dx} = e^x$ — M1

$\displaystyle= \int \frac{1}{4u^2 + 9}\,du$ — A1

$\displaystyle= \frac{1}{4}\int \frac{1}{u^2 + \frac{9}{4}}\,du$ — M1

$= \frac{1}{4} \times \frac{2}{3}\arctan\left(\frac{2u}{3}\right) + c$ — A1

$= \frac{1}{6}\arctan\left(\frac{2}{3}e^x\right) + c$ — A1 **(8)**

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4. *(a)* let $y = \arcsin(2x-1)$ $\therefore \sin y = 2x-1$

$\therefore \cos y \dfrac{dy}{dx} = 2$ M1

$\dfrac{dy}{dx} = \dfrac{2}{\sqrt{1-\sin^2 y}} = \dfrac{2}{\sqrt{1-(2x-1)^2}}$ M1 A1

$= \dfrac{2}{\sqrt{1-(4x^2-4x+1)}} = \dfrac{2}{\sqrt{4x-4x^2}} = \dfrac{1}{\sqrt{x-x^2}}$ A1

(b) $x = \frac{3}{4}$, $y = \arcsin \frac{1}{2} = \frac{\pi}{6}$ B1

$\dfrac{dy}{dx} = \dfrac{1}{\sqrt{\frac{3}{4}-\frac{9}{16}}} = \dfrac{1}{\sqrt{\frac{3}{16}}} = \frac{4}{\sqrt{3}}$ M1

eqn. is $y - \frac{\pi}{6} = \frac{4}{\sqrt{3}}(x - \frac{3}{4})$ M1

$x = 0$ $\therefore y - \frac{\pi}{6} = \frac{4}{\sqrt{3}} \times (-\frac{3}{4})$ M1

$y = \frac{\pi}{6} - \sqrt{3}$ A1 **(9)**

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5. *(a)* $2y\dfrac{dy}{dx} = 4a$ $\therefore \dfrac{dy}{dx} = \frac{2a}{y}$ M1

at P, $\dfrac{dy}{dx} = \frac{2a}{2at} = \frac{1}{t}$ A1

eqn. is $y - 2at = \frac{1}{t}(x - at^2)$ M1

giving $yt = x + at^2$ A1

(b) at Q, $y = 0$ $\therefore x = -at^2$; at R, $x = 0$ $\therefore y = at$ M1 A1

$\therefore M$ is $(-\frac{1}{2}at^2, \frac{1}{2}at)$ A1

(c) grad of $OM = \dfrac{\frac{1}{2}at-0}{-\frac{1}{2}at^2-0} = -\frac{1}{t}$; grad of $OP = \dfrac{2at-0}{at^2-0} = \frac{2}{t}$ M1 A1

OM perp. OP $\therefore \frac{2}{t} \times -\frac{1}{t} = -1$ $\therefore t^2 = 2$ M1 A1 **(11)**

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6. *(a)* $I_n - I_{n-2} = \int \frac{\cos n\theta - \cos(n-2)\theta}{\sin\theta}\,\mathrm{d}\theta$ M1

$= \int \frac{-2\sin(n-1)\theta \times \sin\theta}{\sin\theta}\,\mathrm{d}\theta$ M1 A1

$= \int -2\sin(n-1)\theta\,\mathrm{d}\theta$

$= \frac{2}{n-1}\cos(n-1)\theta$ M1

$\therefore I_n = \frac{2}{n-1}\cos(n-1)\theta + I_{n-2}$ A1

(b) $I_n = [\frac{2}{n-1}\cos(n-1)\theta]_{\frac{\pi}{4}}^{\frac{\pi}{2}} + I_{n-2}$

$I_1 = \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \frac{\cos\theta}{\sin\theta}\,\mathrm{d}\theta = [\ln|\sin\theta|]_{\frac{\pi}{4}}^{\frac{\pi}{2}}$ M1 A1

$= \ln 1 - \ln\frac{1}{\sqrt{2}} = -\ln 2^{-\frac{1}{2}} = \frac{1}{2}\ln 2$ M1 A1

$I_3 = [\frac{2}{2}\cos 2\theta]_{\frac{\pi}{4}}^{\frac{\pi}{2}} + I_1 = -1 - 0 + \frac{1}{2}\ln 2 = \frac{1}{2}\ln 2 - 1$ M1 A1

$I_5 = [\frac{2}{4}\cos 4\theta]_{\frac{\pi}{4}}^{\frac{\pi}{2}} + I_3 = \frac{1}{2} - (-\frac{1}{2}) + \frac{1}{2}\ln 2 - 1 = \frac{1}{2}\ln 2$ M1 A1 **(13)**

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7. *(a)* $ae = \sqrt{3},\ \frac{a}{e} = \frac{4}{\sqrt{3}}\ \therefore ae \times \frac{a}{e} = a^2 = 4$ M1

$a > 0\ \therefore a = 2$ A1

$b^2 = a^2(1-e^2) = a^2 - (ae)^2 = 4 - 3 = 1$ M1

$b > 0\ \therefore b = 1$ A1

(b) $\frac{x^2}{4} + y^2 = 1,\ \frac{1}{2}x + 2y\frac{\mathrm{d}y}{\mathrm{d}x} = 0\ \therefore \frac{\mathrm{d}y}{\mathrm{d}x} = \frac{-x}{4y}$ M1 A1

$A = \int_{-2}^{2} 2\pi y\sqrt{1 + \frac{x^2}{16y^2}}\,\mathrm{d}x$ M1 A1

$= 2\pi\int_{-2}^{2} y\sqrt{\frac{16y^2 + x^2}{16y^2}}\,\mathrm{d}x$

$= 2\pi\int_{-2}^{2} \frac{y}{4y}\sqrt{16(1-\frac{x^2}{4}) + x^2}\,\mathrm{d}x$ M1

$= \frac{\pi}{2}\int_{-2}^{2} \sqrt{16 - 4x^2 + x^2}\,\mathrm{d}x$

$= \frac{\pi}{2}\int_{-2}^{2} \sqrt{16 - 3x^2}\,\mathrm{d}x$ A1

(c) $3x^2 = 16\sin^2\theta,\ x = \frac{4}{\sqrt{3}}\sin\theta,\ \frac{\mathrm{d}x}{\mathrm{d}\theta} = \frac{4}{\sqrt{3}}\cos\theta$ M1 A1

$A = \frac{\pi}{2}\int_{-\frac{\pi}{3}}^{\frac{\pi}{3}} \sqrt{16 - 16\sin^2\theta} \times \frac{4}{\sqrt{3}}\cos\theta\,\mathrm{d}\theta$ M1

$= \frac{\pi}{2}\int_{-\frac{\pi}{3}}^{\frac{\pi}{3}} 4\cos\theta \times \frac{4}{\sqrt{3}}\cos\theta\,\mathrm{d}\theta$

$= \frac{8\pi}{\sqrt{3}}\int_{-\frac{\pi}{3}}^{\frac{\pi}{3}} \cos^2\theta\,\mathrm{d}\theta$ A1

$= \frac{4\pi}{\sqrt{3}}\int_{-\frac{\pi}{3}}^{\frac{\pi}{3}} 1 + \cos 2\theta\,\mathrm{d}\theta$ M1

$= \frac{4\pi}{\sqrt{3}}[\theta + \frac{1}{2}\sin 2\theta]_{-\frac{\pi}{3}}^{\frac{\pi}{3}}$ A1

$= \frac{4\pi}{\sqrt{3}}[\frac{\pi}{3} + \frac{\sqrt{3}}{4} - (-\frac{\pi}{3} - \frac{\sqrt{3}}{4})] = \frac{4\pi}{\sqrt{3}}(\frac{2\pi}{3} + \frac{\sqrt{3}}{2}) = \frac{8}{9}\pi^2\sqrt{3} + 2\pi$ M1 A1 **(18)**

Total **(75)**

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Performance Record – FP2 Paper H

Question no.	**1**	**2**	**3**	**4**	**5**	**6**	**7**	Total
Topic(s)	rad. of curv.	eqn. in hyp. fns.	integr. hyp. fns	diff. inv. trig.	parabola, tangent	reduction formula	ellipse, surface area	
Marks	8	8	8	9	11	13	18	75
Student								

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